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Question Paper Code : 11632

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2026.

Third Semester

Electrical and Electronics Engineering

MA3303 — PROBABILITY AND COMPLEX FUNCTIONS

(Regulations 2021)

Time : Three hours

Maximum : 100 marks

(Use of Statistical tables are permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If A and B are two events such that $P(A|B) = 0.4$, $P(B) = 0.8$, $P(A) = 0.5$, are the events A and B independent?
2. Let the random variable X have a continuous uniform distribution on the interval $0 \leq x \leq 15$. Determine the mean and variance of X .
3. Random variables X and Y have joint PDF
$$f_{X,Y}(x, y) = \begin{cases} \frac{1}{2}; & -1 \leq x \leq 1, -1 \leq y \leq 1, \\ 0, & \text{otherwise} \end{cases}$$
 then find $f_X(x)$.
4. State the central limit theorem.
5. For what values of a, b, c the function $f(z) = (x - 2ay) + i(bx - cy)$ is analytic.
6. Find the fixed points of $w = \frac{1+z}{1-z}$.
7. State the Cauchy's integral formula.
8. Find the residue of $f(z) = \frac{\sin z}{z^3}$ at $z = 0$.
9. Find the Wronskian of $y'' + 9y = \sin 2x$
10. Find the particular integral of $(D^2 + D + 1)y = x$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) State and prove Baye's theorem. (6)
- (ii) A factory has three machines, A, B and C, producing parts. Machine A produces 50% of the parts, Machine B produces 30%, and Machine C produces 20%. The defect rates for each machine are : A (2%), B(3%), and C (5%). If a randomly selected part is defective, what is the probability that it was produced by Machine A? (10)

Or

- (b) (i) The number of telephone calls that arrive at a phone exchange is often modelled as a Poisson random variable. Assume that on the average there are 15 calls per hour. (6)
- (1) What is the probability that there are exactly 5 calls in one hour?
- (2) What is the probability that there are 3 or less calls in two hours?
- (3) What is the probability that there are exactly 5 calls in 30 minutes?
- (ii) The IQ level X , of a randomly selected individual is often supposed to follow a Gaussian distribution with mean 100 and standard deviation 15. (10)
- (1) Find the probability that an individual has an IQ level is below 140
- (2) Find the probability that an individual has an IQ level is above 150
- (3) Find the probability that an individual has an IQ level is below 120 and 130
- (4) Find the value of x , such that $P(X < x) = 0.99$.

12. (a) Random variables X and Y have the joint PMF

$$P_{X,Y}(x, y) = \begin{cases} kxy; & x = 1, 2, 3; y = 1, 2, 3, \\ 0 & \text{otherwise} \end{cases}$$

- (i) What is the value of the constant k ?
- (ii) What is $P[Y \geq X]$ and $P[x \leq 2]$?
- (iii) Find the correlation coefficient $\rho_{X,Y}$.

Or

- (b) Random variables X and Y have the joint PDF

$$f(x, y) = \begin{cases} k(1 - e^{-x})(1 - e^{-y}); & x > 0; y > 0, \\ 0 & \text{otherwise} \end{cases}$$

Then find

- (i) value of k
- (ii) $f_x(x, y)$ and $f_y(x, y)$
- (iii) $P[X + Y < 3]$
- (iv) $P[1 < X < 3, 1 < Y < 2]$
- (v) Are X and Y independent?

13. (a) (i) Derive the Cauchy-Riemann equations for $f(z)$ to be analytic. (8)
- (ii) Determine a so that the given function $u(x, y) = \cos ax \cosh 2y$ is harmonic and find its harmonic conjugate. Also construct an analytic function $f(z) = u(x, y) + iv(x, y)$. (8)

Or

- (b) (i) Find the linear fractional transformation that maps $z_1 = 0, z_2 = 1, z_3 = \infty$ onto $w_1 = -1, w_2 = -i, w_3 = 1$ respectively. (8)
- (ii) Show that the transformation $w = \frac{1}{z}$ transforms all circles and straight lines in the z -plane into circles or straight lines in the w -plane. (8)

14. (a) State and prove Cauchy residue theorem. Apply it evaluate $\oint_C \frac{(z - 23) dz}{z^2 - 4z - 5}$, where C the circle $|z - 2 - i| = 3.3$ counter clockwise.

(8+8=16)

Or

- (b) (i) Evaluate $\oint_C \frac{z dz}{z^2 + 4z + 3}$, where C is the circle with center -1 and radius 2 . (8)
- (ii) Find all the Laurent series for the function $f(z) = \frac{-2z + 3}{z^2 - 3z + 2}$ about $z_0 = 0$ and discuss about the region of convergence. (8)

15. (a) (i) Solve the differential equation $(x^2 D^2 - 2xD - 4)y = x^2 + 2 \log x$. (8)
- (ii) Solve the differential equation $(D^2 + 1)y = x \sin x$, by the method of variation of parameters. (8)

Or

- (b) (i) Solve $(D^2 + 4)y = 5e^x \cos x$ using Method of undermined co-efficient. (8)
- (ii) Solve $\frac{dx}{dt} + 2y = 5e^t$, $\frac{dy}{dt} - 2x = 5e^t$, given that $x = -1$ and $y = 3$ at $t = 0$. (8)